SPECTRAL ACTION, NONCOMUTATIVE GEOMETRY and PARTICLE PHYSICS

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Contents:

- Noncommutative Geometry and Spectral Action
- Weak-field approximation and the Standard Model
- High-energy expansion

Based on a joint works: B.Iochum, C. Levi and DV, CMP (2012) and M.Kurkov, F.Lizzi and DV, PLB (2014)

Noncommutative Geometry (Alain Connes)

Basic object is the *spectral triple* $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ consisting of
- an associative, possibly non-commutative algebra $\mathcal{A}$ (commutative case: algebra of smooth or continuous functions).
- a Hilbert space $\mathcal{H}$ where $\mathcal{A}$ acts by bounded operators (commutative case: sections of a spin bundle).
- a s.a. "Dirac operator" $\mathcal{D}$ defined on $\mathcal{H}$ satisfying:

$[\mathcal{D}, a]$ is bounded for all $a \in \mathcal{A}$ (This implies in the commutative case that $\operatorname{ord} \mathcal{D} \leq 1$)
$(\mathcal{D} + i)^{-1}$ is compact (This implies that $\mathcal{D}$ is an elliptic 1st order DO).

Reconstruction Theorems:

These ingredients are sufficient to recover all geometric structures in the commutative case.
Th.: If $\mathcal{A}$ is unital commutative, there is a compact Riemannian manifold M and a vector bundle V over M s.t. $\mathcal{D}$ is a Dirac type operator on $\Gamma(V)$.
Remark: If one requires some additional structures like chirality and conjugations, then M is a spin manifold.

Spectral Action(Chamseddine and Connes)

Physical (bosonic) fields, like the metric, Yang-Mills field, Higgs, enter the game through the Dirac operator. Am action for these fields has to be an invariant functional. The most natural choice is to postulate the Spectral Action to be

$$S = \mathrm{Tr}\left(f(\mathcal{D}^2/\Lambda^2)\right)$$

where f is a real function, which is restricted by the assumption that the trace exists. Λ is a cut-off needed to make the argument dimensionless.

Reminds the path integral over low energy region (Andrianov, Novozhilov, DV,...)!

Can this be calculated?

Weak-field expansion

Suppose, that f is a Laplace transform,

$$f(z) = \int_0^\infty dt e^{-tz} \varphi(t)$$

Then, taking the trace,

$$S = \int_0^\infty dt K(t/\Lambda^2, \mathcal{D}^2) \varphi(t)\,,$$

where

$$K(s, \mathcal{D}^2) = \mathrm{Tr}\left(e^{-s\mathcal{D}^2}\right)$$

is the heat kernel, which admits an asymptotic expansion

$$K(s, \mathcal{D}^2) = \sum_{k=0} s^{-m+k} a_{2k}$$

as $s \to +0$, with $2m$ being dimension of the spectral triple.

For the action:

$$S \sim \sum_k \Lambda^{2(m-k)} \varphi_{2k} a_{2k}(\mathcal{D}^2)$$

with

$$\varphi_{2k} = \int_0^\infty dt\, t^{-m+k} \varphi(t)$$

For "reasonable" choices of $\mathcal{D}$ the heat kernel coefficients are well known. For the standard "commutative" Dirac operator:
$a_0 \sim \int \sqrt{g}$ – the cosmological constant;
$a_2 \sim \int \sqrt{g} R$ – the Einstein-Hilbert action;
$a_4 \sim \int \sqrt{g} F_{\mu\nu} F^{\mu\nu}$ – the Yang-Mills action, etc.

The Standard Model:

Almost commutative spectral triple: $\mathcal{A} = C^\infty(\mathbb{R}^4) \times \mathcal{A}_{sm}$, where $\mathcal{A}_{sm} = \mathbb{C} \oplus \mathbb{H} \oplus \mathrm{Mat}_3(\mathbb{C})$ is a finite dimensional matrix algebra.
Gauge group consists of unitary elements of $\mathcal{A}$ and thus coincides with $U(1) \times SU(2) \times U(3)$. Since $\mathcal{H}$ has to be a representation space of $\mathcal{A}$ rather than that of the gauge group, only defining representations are allowed.
The Higgs field appears in this approach as a component of the connection (in the "internal direction).
The action for fermions reads

$$S_f = \langle \psi, \mathcal{D}\psi \rangle$$

Obviously, $\mathcal{D}$ is fixed in the Standard Model.

Let us compute the spectral action for the same $\mathcal{D}$. First 3 terms of the low-energy expansion reproduce nicely the bosonic part of Standard Model actions except that:

- The Higgs field is too heavy at about 170GeV. May be lowered in various extensions of the model.
- The spectral action predicts a unification point where all gauge couplings become equal. This may be resolved by including the next expansion term (Devastato, Lizzi, Varcárcel, DV).
  The spectral action principle is quite successful in describing the Standard Model.

Beyond the weak-field expansion

Let us take $f(z) = e^{-z}$ (as a typical example) and

$$\mathcal{D} = \not{D} + \gamma_5 \phi, \qquad \not{D} = i\gamma^\mu(\nabla_\mu^{LC} + iA_\mu)$$

Then, $S = K(\mathcal{D}^2, 1/\Lambda^2)$. One can use the Barvinsky-Vilkovisky covariant perturbation theory for the heat kernel to compute the spectral action to quadratic order in bosonic fields to the leading order in the expansion in $\frac{\Lambda^2}{-\partial^2}$ (i.e., in the large momentum expansion). In the 4D case, it reads:

$$K(\mathcal{D}^2, s = 1/\Lambda^2) \simeq \frac{\Lambda^4}{(4\pi)^2} \int d^4x \left[-\tfrac{3}{2} h_{\mu\nu} h_{\mu\nu} + 8\phi \frac{1}{-\partial^2} \phi + 8 F_{\mu\nu} \frac{1}{(-\partial^2)^2} F_{\mu\nu} \right]$$

[for a generic f just the numerical coefficients differ].
Main observations: (i) There were some miraculous cancellation of the leading terms in the general BV fomula. (ii)
There are NO terms with positive powers of derivatives.
THERE IS NO PROPAGATION OF BOSONIC FIELDS AT HIGH MOMENTA.

Discussion

I. Democracy High momentum modes are suppressed in the fermionic sector by the cut-off function f. Then, high momentum bosonic modes are suppressed dynamically in the Spectral Action.
II. Quantum Gravity. Since there are no high momentum gravitons, the problem of perturbative nonrenormalizability of quantum gravity is no longer relevant. Unfortunately, we still do not know what the correct quantum gravity looks like.
III. Cosmology. Usually, Λ is taken to be $10^{14} - 10^{16}$GeV, which is several orders of magnitude below the Planck mass. There is a possibility that the Spectral Action principle will have testable cosmological predictions.